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Whiteness formulas: a selection

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Experience in assessing whiteness and industrial requirements is considered in the selection of whiteness formulas. The problems set by the photometry of fluorescent samples and the colorimetric prerequisites delimit the choice. A standard whiteness formula of neutral hue preference is proposed. Two supplementary whiteness formulas of green and red hue preference, respectively, are added. The feasibility of a complete characterization of white samples by whiteness, tint, and luminance factor is discussed.

Introduction

Whiteness measurement is replacing visual assessment, which is impaired by individual preference. However, whiteness formulas of various structures and with different properties are presently being applied side by side. Instrumental whiteness assessment will be superior only if the uncertainties of measurement and of evaluation are overcome. The aim of this paper is to propose some whiteness formulas for general use.

Fundamental Aspects

In selecting whiteness formulas, different problems from those experienced with color difference formulas are encountered. In assessing color differences, most observers agree at least qualitatively in the valuation of samples. Apart from the quantitative aspect of acceptability and perceptibility, reference data sets can be used for testing color difference formulas as to uniform chromaticity.

No general agreement exists in the valuation of whiteness. Contradictory assessments are caused by individual differences in hue preference as well as by preconceptions in various trades. It is unlikely, therefore, that the selection of a single whiteness formula would be satisfactory.

Conversely, uniform chromaticity is no problem in the assessment of whiteness, since white colors cover only a small part of the color space. Experience in producing white scales has shown that the chromaticities of uniformly spaced samples turn out to be approximately equidistant in the x,y chromaticity chart.

An additional problem of metamerism arises. The intensity of fluorescence determining the appearance of white fluorescent samples depends on the spectral power distribution of the illumination, especially in the uv wavelength range. Differences between visual assessment and measurement as well as differences between measurements carried out with various photometers are usually caused by differences in the spectral power distribution of the illumination.

Furthermore, it must be noted that spectral and photometric errors in color measurement, which only produce second-order effects on color difference, directly influence the evaluation of whiteness. The spectral errors incurred in the measurement of fluorescent whites with colorimeters may cause considerable errors in chromaticity. Both tint and whiteness are thus falsified though the influence on whiteness is relatively small as compared with the effect of variations in the uv contents of the illumination.

Present Practice

Realization of the limitations outlined had various consequences:

(1) Whiteness determined by colorimetric measurements with the aid of a whiteness formula is no longer considered as an absolute quantity. It is accepted that its value depends on the model of photometer used, on the state of the illumination, and on the properties of the whiteness formula employed. Only whiteness differences calculated for samples measured with the same photometer at the same time are considered significant.

(2) Since it has hitherto not been possible to manufacture a stable source producing the relative power distribution of illuminant D65 with sufficient accuracy at least in the 320–550-nm wavelength range, methods were developed for converting to illuminant D65, the spectral radiance factors that are measured with a source of deviating but known relative spectral power

distribution.[1-3] These methods are valuable for the calibration of reference samples, etc., but they are too cumbersome for routine work.

(3) An approximation to constant relative spectral power distribution was attained by a device for controlling the relative uv contents of the sample irradiation in photometers.[4] With an adjustable uv filter, the excitation of the fluorescence of a stable white reference sample is attenuated sufficiently to keep the tristimulus values of this sample constant. The long term reproducibility of the measurements lies within the threshold limits of perceptible whiteness differences.

(4) A simple procedure was devised which can be applied with any type of photometer or colorimeter.[5,6] It is based on stable fluorescent samples such as the CIBA-GEIGY Plastic White Scale. With the visually determined nominal whiteness of the samples and with their tristimulus values measured with the photometer to be calibrated, the factors of a whiteness formula can be optimized by linear regression for the desired hue preference. The procedure is suitable for routine work. It can easily be repeated as often as required by the aging of the source and of the integrating sphere. It must be pointed out, however, that the spectral coefficients of absorption and of emission of the fluorescent whitening agent in the reference samples act as weighting functions. The same applies to the method of approximation described in the previous paragraph.

Choice of the Type of Whiteness Formula

Regardless of the problems presented by the colorimetry of fluorescent samples, attempts are being made to standardize the calculation of whiteness. Most formulas used at present assess a sample as all the whiter, the lighter, and the bluer it actually is. If applied to colored samples, the calculated whiteness values obviously are meaningless. However, these formulas satisfactorily characterize the appearance of commercial whites. They fail in assessing tinted samples with chromaticities situated on the borders of the range of white colors. For dealing with such extreme cases, special formulas were developed. In the chromaticity chart, they produce families of ellipses as equiwhiteness lines that should be centered on the yet unknown preferred white.[5,7-10] Under normal viewing conditions, this point is not reached or even bypassed, since the relatively small uv contents of natural daylight and of most sources used for illumination limit the excitation of fluorescence. For routine work, therefore, formulas of this type need not be considered.

A formula based on MacAdam's geodesic chromaticity diagram and tested by visual assessment of two sample sets was compared with equivalent Yxy, Lba, and BGA formulas.[5] With all formulas, correlation coefficients of 0.99 for neutral hue preference and ⩾0.985 for red and green preference were obtained. In view of this and of the experience mentioned earlier with uniformly spaced white scales, it appears not worthwhile

to consider for routine work the construction of whiteness formulas in the CIE L*u*v* or L*a*b* color spaces. For both these reasons, formulas linear in XYZ or Yxy are considered only in this selection.

Formerly, most instrumental whiteness assessments were carried out with colorimeters. Thus various BGA type whiteness formulas

$$W = \beta \cdot B + \gamma \cdot G + \alpha \cdot A, \tag{1}$$

with

$$B = 100 \cdot Z/Z_0,$$

$$G = 100 \cdot Y/Y_0,$$

$$A = (1 + g) \cdot 100 \cdot X/X_0 - g \cdot 100 \cdot Z/Z_0$$

$$g = 0.234 \text{ for illuminant D65, and}$$

$$\beta + \gamma + \alpha = 1 \tag{2}$$

were set up and widely used. For this investigation BGA formulas were, therefore, taken as a starting point and for reference. Subsequently, they were compared with similarly structured Z/Z_0, Y/Y_0, and X/X_0 formulas

$$W = \beta_T \cdot Z/Z_0 + \gamma_T \cdot Y/Y_0 + \alpha_T \cdot X/X_0, \tag{3}$$

with

$$\beta_T + \gamma_T + \alpha_T = 100, \tag{4}$$

and with formulas of the Yxy type of equivalent properties

$$W = 100 \cdot Y/Y_0 + p (x - x_0) + \sigma (y - y_0), \tag{5}$$

which are used in the form

$$W = Y + p \cdot x + \sigma \cdot y - C_W; \quad C_W = p \cdot x_0 + \sigma \cdot y_0$$

for routine applications. X_0, Y_0, Z_0, x_0, and y_0 denote the tristimulus values and the chromaticity coordinates of the perfect diffuser.

Choice of the Properties of the Whiteness Formulas

Any whiteness formula defines surfaces of constant whiteness in the color space. The factors $(\beta\gamma\alpha)$, $(\beta_T\gamma_T\alpha_T)$, and $(\rho\sigma)$, respectively, imply the orientation and the spacing of these surfaces. Parameters suitable for characterizing the equiwhiteness surfaces in any point Yxy were developed.[5,11] They are based on luminous reflectance Y and on transformed chromaticity coordinates, i.e., colorimetric saturation

$$s = (x_0 - x) \cos\eta + (y_0 - y) \sin\eta, \tag{6}$$

and, perpendicular to it, colorimetric tint

$$f = (x_0 - x) \sin\eta - (y_0 - y) \cos\eta. \tag{7}$$

x_0 and y_0 are the coordinates of the perfect diffuser, and η is the angle subtended by a reference dominant wavelength and the x axis. Since variations in whiteness of neutral tint caused by changes in colorimetric saturation are observed in the range of dominant wavelengths of approximately 465–475 nm, $\lambda_d = 470$ nm was selected as the reference dominant wavelength.
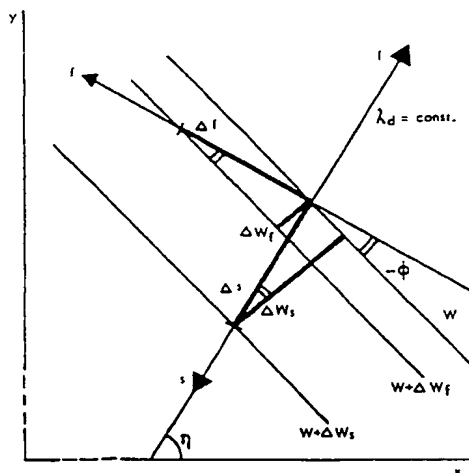
Fig. 1. Definition of colorimetric saturation s, colorimetric tint f, and hue preference angle ϕ.

Table I. Characteristic Parameters of Some Well-Known Whiteness Formulas, Illuminant D65 and CIE 1931 2° Standard Observer

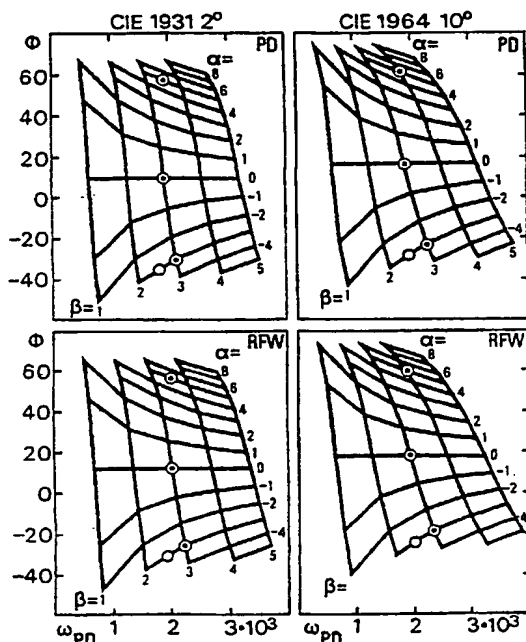| Formula	Representative fluorescent white		Perfect diffuser		Author
	ω	ϕ	ω	ϕ	
$W = B$	556	11	638	9	(TAPPI)
$W = B + G - A$	603	−26	705	−30	Croes
$W = 2B - A$	984	−9	1343	−13	Stephansen
$W = 3B + G - 3A$	1319	−26	2115	−30	Berger
$W = 4B - 3G$	1468	11	2553	9	Taube
$W = L - 3b$	1964	11	2680	9	Hunter
$W = L - 3b + 3a$	2298	55	3384	56	Stensby

Fig. 2. Correlation of the parameters ω_{PD} and ϕ with the factors α, β, and $\gamma = 1 - \alpha - \beta$ of BGA type whiteness formulas (1). The selected formulas 1.3, 2.3, 3.7, and 3.8 are marked ⊙; formula 2.4 is marked O.
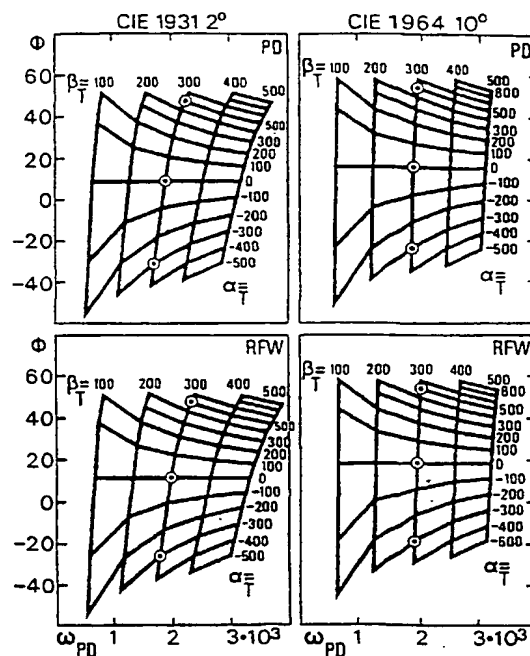

Fig. 3. Correlation of the parameters ω_{PD} and ϕ with the factors α_T, β_T, and $\gamma_T = 100 - \alpha_T - \beta_T$ of Z/Z_0, Y/Y_0, and X/X_0 type whiteness formulas (3). The selected formulas 1.2, 2.2, and 3.4 are marked ⊙.
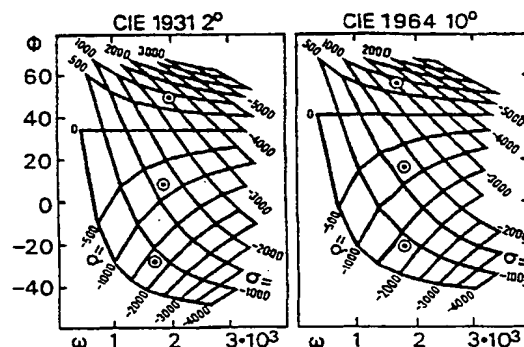

Fig. 4. Correlation of the parameters ω and ϕ with the factors ρ and σ of Yxy type whiteness formulas (5). The selected formulas 1.1, 2.1, and 3.1 are marked ⊙.

As equiwhiteness surface can be characterized in any point Yxy by its slope ω in the plane of the reference dominant wavelength

$$\omega = (\partial W / \partial s) / (\partial W / \partial Y), \tag{8}$$

and by its angle ϕ with respect to the perpendicular of the reference dominant wavelength

$$\phi = -\arctan[(\partial W / \partial f) / (\partial W / \partial s)] \tag{9}$$

(see Fig. 1). These characteristic parameters were calculated for a variety of whiteness formulas. The correlation with the numerical factors was displayed in diagrams.[5] In Table I, the characteristic parameters of some well-known whiteness formulas are shown for a representative fluorescent white ($Y = 90.00$, $s =$

0.0300, and $\lambda_d = 470$ nm) and for the perfect diffuser. Both the slope ω and the hue preference angle ϕ vary over a wide range.

The round robin test organized by the CIE TC-1.3 Subcommittee on Whiteness[12] had shown that there are individuals with extreme hue preference exceeding either the green preference of the Croes and Berger formulas or the red preference of the Stensby formula. No conclusions were drawn as to the variations of ω. However, the round robin test definitely showed that it will be impossible to reproduce the hue preferences of a significant fraction of visual assessment by a single whiteness formula.

This is confirmed by industrial experience. A range of hue preferences of about $\phi \sim -30°$ to $\phi \sim 60°$ appears necessary and sufficient to cover most instrumental whiteness assessments. Little is known about the range of the slope ω. Most assessments are concerned with samples of similar luminance factors but differing in fluorescence intensity and in tint. Preliminary unpublished assessments were carried out with special samples of constant tint which differ in luminance factor and in fluorescence intensity. Values of $\omega \sim 700$ slightly exceeding the slope of the MacAdam limit for dominant wavelength 470 nm up to infinity for $(\partial W/\partial Y) \rightarrow 0$ were found. The mean of thirty subjects yielded $\omega \sim 2300$. This is in the range shown for the formulas in Table I. Since the value of ω is not critical for most routine assessments, it seems acceptable to select an average value of $\omega \sim 1800$.

It is usual to define the perfect diffuser (PD) as the reference point of the whiteness scale $W_{PD} = 100$ and $(\partial W/\partial Y)_{PD} = 1$ as the unit of whiteness. For formulas (1) and (3), these conditions are met by the restrictions (2) and (4), respectively.

Choice of the Factors of the Whiteness Formulas

Having selected the desired properties of the whiteness formulas, the numerical values of the factors producing such properties must be determined. The interrelationship between the factors and the properties of whiteness formulas of types (1), (3), and (5) was computed[11] and is shown in Figs. 2–4. Every point in these diagrams represents one whiteness formula with the properties ω and ϕ and the factors (β,γ,α), $(\beta_T,\gamma_T,\alpha_T)$, and (ρ,σ), respectively. For formulas of the Yxy type (5), the values of ω and ϕ are independent of the luminance factor and of the chromaticity, since the surfaces of equiwhiteness are defined by parallel equidistant planes.

For formulas (1) and (3), $(\partial^2 W/\partial Y^2) = 0$ is the only vanishing second derivative. Therefore, ω and ϕ depend on chromaticity and on the luminance factor (see Table I). The significantly reduced values of $\omega = (\partial W/\partial s)/(\partial W/\partial Y)$ for the representative fluorescent white (RFW) as compared with ω for the perfect diffuser (PD) are not caused by a decrease of $(\partial W/\partial s)$ but by a marked increase of $(\partial W/\partial Y)$. The relatively high positive values of $(\partial^2 W/\partial Y \partial s)$ produced by formulas (1) and (3) are not confirmed by experimental evidence, however, and it seems unlikely that the eye would per-

ceive whiteness differences more sharply the more closely the stimuli approach the preferred white (see Ganz,[5] Fig. 5). Considering this particularity, the influence of the variation of $(\partial W/\partial Y)$ on ω was canceled by choosing $\omega_{PD} = \omega \cdot (\partial W/\partial Y)/(\partial W/\partial Y)_{PD}$ as abscissa in Figs. 2 and 3, thus facilitating the comparison of the diagrams for the perfect diffuser and for the representative fluorescent white.

Diagrams are shown for illuminant D65 and both CIE 1931 2° and 1964 10° standard observers. The differences are caused by the deviation of the slopes of dominant wavelengths in the vicinity of 470 nm. They have only to be accounted for in special cases, e.g., for keeping constant the whiteness of neutral tint if calculated by whiteness formulas of different hue preference. The slightly increased green preference of all formulas applied to colorimetric data for the 2° observer as compared with the 10° observer is not significant.

In view of the errors introduced during measurement by deviations of the relative uv content from that of illuminant D65, round figures only should be selected as factors in the whiteness formulas. Thus a deceptive appearance of accuracy is avoided. Precise figures should be reserved for factors periodically calibrated with white reference samples.

Figures 2 and 3 show that the value of ω_{PD} is determined primarily by the value of the factors β and β_T, respectively. Values of $\beta = 3$ and $\beta_T = 300$ yield roughly the desired value of ω_{PD}. Neutral hue preference $(\phi \sim 15°)$ is produced by values of $\alpha = 0$ and $\alpha_T = 0$, with values of $\gamma = -2$ and $\gamma_T = -200$ according to formulas (2) and (4), see Table II, formulas 1.3 and 1.2. The two formulas are identical since $B = 100 \cdot Z/Z_0$ and $G = 100 \cdot Y/Y_0$. They have the same neutral hue preference as the Taube, Hunter, and (TAPPI) formulas, and the value of their ω_{PD} approximately corresponds with the mean of the three formulas referred to (see Table I).

The whiteness value of about 145 determined for the reference fluorescent white by both observers is taken as the reference value for selecting the factors of all other formulas. Formula 1.1 (Fig. 4) is practically equivalent to 1.2 and 1.3 but avoids the undesirable increase of $\partial W/\partial Y$ for the RFW.

Whiteness with a green hue preference is calculated using formulas 2.1 to 2.4. Formula 2.2 is not identical with the Berger formula 2.3, since $A \neq 100 \cdot X/X_0$. If the same value W_{RFW} of whiteness for the RFW as calculated by formula 1.3 should be obtained, formula 2.4 is recommended rather than formula 2.3.

The hue preference of formulas 3.1–3.8 corresponds with the red hue preference of the Stensby formula. Whereas small differences only arise between the values of W_{RFW} calculated for the 2° and 10° observers with green hue preference, the divergences for red hue preference become appreciable. If close agreement of W_{RFW} with the values determined by formula 1.1 is required, formula 3.2 should be used for the 10° observer, and formula 3.3 should be used for the 2° observer rather than formula 3.1. Formulas 3.4 and 3.8 are available for the 10° observer as are formulas 3.5 and

| | | CIE 1931 2° | | | | | | CIE 1964 10° | | | | | |
| | | Representative fluorescent white | | | | Perfect diffuser | | Representative fluorescent white | | | | Perfect diffuser | |
Whiteness formula		W	$\partial W/\partial Y$	$\partial W/\partial s$	Φ	$\partial W/\partial s$	Φ	W	$\partial W/\partial Y$	$\partial W/\partial s$	Φ	$\partial W/\partial s$	Φ
Neutral hue preference													
1.1	$W = Y - 800(x - x_0) - 1700(y - y_0)$	145.6	1.000	1852	9.6	1852	9.6	144.0	1.000	1800	16.6	1800	16.6
1.2	$W = 300Z/Z_0 - 200Y/Y_0$	145.9	1.621	2013	11.4	1914	9.2	144.1	1.601	1934	18.2	1868	16.1
1.3	$W = 3B - 2G$	145.9	1.621	2013	11.4	1914	9.2	144.1	1.601	1934	18.2	1868	16.1
Green hue preference													
2.1	$W = Y - 1700(x - x_0) - 900(y - y_0)$	141.3	1.000	1709	−27.3	1709	−27.3	144.1	1.000	1804	−20.3	1804	−20.3
2.2	$W = 300Z/Z_0 + 100Y/Y_0 - 300X/X_0$	140.0	1.556	1802	−25.9	1714	−30.2	143.0	1.589	1895	−18.6	1830	−23.0
2.3	$W = 3B + G - 3A$	151.7	1.686	2224	−25.9	2115	−30.2	155.4	1.727	2338	−18.6	2259	−23.0
2.4	$W = 2.5B + 1.5G - 3A$	142.4	1.582	1888	−31.3	1796	−35.5	146.4	1.626	2016	−23.9	1947	−28.4
Red hue preference													
3.1	$W = Y + 800(x - x_0) - 3000(y - y_0)$	150.2	1.000	2006	49.7	2006	49.7	141.1	1.000	1702	56.8	1702	56.8
3.2	$W = Y + 700(x - x_0) - 3100(y - y_0)$	154.4	1.000	2145	47.5	2145	47.5	145.3	1.000	1843	54.6	1843	54.5
3.3	$W = Y + 900(x - x_0) - 2900(y - y_0)$	146.0	1.000	1867	52.1	1867	52.1	136.8	1.000	1561	59.1	1561	59.1
3.4	$W = 300Z/Z_0 - 700Y/Y_0 + 500 X/X_0$	155.6	1.729	2365	47.1	2249	47.9	145.9	1.621	1999	54.1	1931	54.8
3.5	$W = 250Z/Z_0 - 650Y/Y_0 + 500X/X_0$	146.3	1.626	2030	50.7	1930	51.6	136.9	1.521	1677	57.7	1620	58.5
3.6	$W = 3B - 7G + 5A$	136.1	1.512	1662	60.1	1580	61.2	125.2	1.392	1260	67.3	1217	68.2
3.7	$W = 3.5B - 7.5G + 5A$	145.4	1.616	1997	56.0	1899	57.1	134.3	1.492	1582	63.1	1529	63.9
3.8	$W = 4B - 8G + 5A$	154.7	1.719	2333	52.4	2218	53.4	143.3	1.592	1905	59.4	1840	60.2

	Tint formula	Neutral λ_d(nm)	T	$\partial f/\partial T$	T	$\partial f/\partial T$	Neutral λ_d(nm)	T	$\partial f/\partial T$	T	$\partial f/\partial T$
4.1	$T = -950(x - x_0) + 750(y - y_0)$	473.6	−2.2	0.00083	0.0	0.00083	467.3	2.2	0.00083	0.0	0.00083
4.2	$T = - 900(x - x_0) + 800(y - y_0)$	476.2	−4.3	0.00083	0.0	0.00083	469.9	0.1	0.00083	0.0	0.00083
4.3	$T = -1000(x - x_0) + 700(y - y_0)$	470.2	−0.1	0.00082	0.0	0.00082	463.6	4.3	0.00082	0.0	0.00082

Table III. Whiteness $W_{1.1}$ of the Samples of the CIBA-GEIGY Plastic White Scale, Computed with Whiteness Formula 1.1 for Illuminant D65, Rounded to the Next Even Integer

Sample	$W_{1.1}$
1	40
2	52
3	60
4	70
5	78
6	88
7	96
8	106
9	116
10	126
11	132
12	144

3.7 for the 2° observers. Formula 3.6 is not recommended.

Adjustment of the Factors to the Actual UV Contents in the Measurement

If the differences in whiteness caused by deviations of the relative uv contents during measurement are troublesome, the factors of whiteness formulas of the Yxy type can be adjusted by the procedure[5] mentioned before. In order to make the CIBA-GEIGY Plastic White Scale usable with the recommended formulas, the whiteness of the samples shown in Table III was calculated with formula 1.1 using colorimetric data evaluated with measured spectral radiance factors that were converted[1] to illuminant D65. The values (ρ/σ) for calculating the regression with formulas 1.1, 2.1, and 3.1 are 0.4706, 1.8889, and −0.2667, respectively. Unless the measurement of the reference samples is periodically repeated and the factors are readjusted accordingly, the factors presented as round figures in Table II should always be used.

The factors of whiteness formulas (1) and (3) could also be adjusted similarly.[6] However, Levene and Knoll have shown that the precision of the factor adjustment is smaller than for the recommended formulas of the Yxy type (5). Therefore, whiteness determinations with adjusted formulas (1) and (3) would be less accurate.

Choice of a Tint Formula

The whiteness value does not characterize a white sample completely. It is customary to assess the tint visually and designate it by coloristic terms, e.g., RR, R, B, G, and GG for reddish, neutral (blue), and greenish whites. Based on visual assessments of the paper samples prepared by Berger[13] for the CIE TC-1.3 Subcommittee on Whiteness, a tentative formula for tint

$$T = \mu(x - x_0) + \nu(y - y_0) \qquad (10)$$

was set up.[5] It is used in the form

$$T = \mu \cdot x + \nu \cdot y - C_T; \quad C_T = \mu \cdot x_0 + \nu \cdot y_0,$$

yielding values of $T < 0$ for red, and $T > 0$ for green tints.

The formula was extensively tested in industrial laboratories and found to be superior to visual assessments and to other colorimetric evaluations, e.g., dominant wavelength. Though suitable to characterize the tint of samples of high whiteness, dominant wavelength is inadequate for evaluating tint in the vicinity of the chromaticity of the perfect diffuser. Samples of equally perceived tint may be located at quite different dominant wavelengths and vice versa.

The lines of constant tint were found to run approximately parallel to dominant wavelength 470 nm. A bandwidth $(\partial f/\partial T)$ of 0.00080–0.00085 units of the chromaticity chart turned out to correspond with the mean of coloristic tint assessments. Since the slope of dominant wavelength 470 nm differs considerably in the chromaticity charts for the 2° and 10° observers, the use of a formula with the same factors for both observers is not recommended. The tint of the neutral representative fluorescent white calculated by formula 4.1 with 10° colorimetric data would be $G2$ (two units green, $T_{RFW} = 2.2$) and conversely $R2$ (two units red, $T_{RFW} = -2.2$) with 2° colorimetric data. Formulas 4.2 and 4.3 yield the correct neutral tint ($T_{RFW} = \pm 0.1$) for 10° and 2° colorimetric data, respectively. The influence of the relative uv contents in the measurement on the evaluation of tint is negligible.

Contributions of Luminance Factor and of Chromaticity to Whiteness

The third variable for characterizing whites is the luminance factor Y. With the Yxy formulas as well as with many other carefully investigated formulas (Thielert and Schliemann,[9] Vaeck,[10] and Anders and Daul[14]), $(\partial W/\partial Y)$ is constant throughout the range of whites and not only on the achromatic axis as with the BGA and Z/Z_0, Y/Y_0, and X/X_0 type formulas. With the former, the contributions of luminance factor and of chromaticity to whiteness are additive.

The kind of whiteness can then be characterized by the contribution of chromaticity

$$C = W - Y = \rho(x - x_0) + \sigma(y - y_0), \qquad (11)$$

which is $C > 0$ and $W > Y$ for bluish whites, $C < 0$ and $W < Y$ for yellowish whites.

Range of Validity of the Whiteness Formulas

To prevent application of whiteness formulas to colored samples which would be meaningless, boundaries in the color space were proposed.[14] The following limits for white colors are selected;

$$Y > 70,$$

$$W > 40,$$

$$-6 < T < 6.$$

No limit is proposed on the blue side, since the location of the preferred white, which also depends on the surround, is not sufficiently known and rarely reached.

Conclusion

The formula

$$W = Y - 800(x - x_0) - 1700(y - y_0)$$

is proposed as a standard whiteness formula of neutral hue preference. Two supplementary whiteness formulas, one of green hue preference:

$$W = Y - 1700(x - x_0) - 900(y - y_0),$$

and one of red hue preference:

$$W = Y + 800(x - x_0) - 3000(y - y_0),$$

apparently are required to cover all evaluations. The three formulas can be used with colorimetric data evaluated for standard illuminant D65 and both CIE 1931 2° and CIE 1964 10° observers. Two standard tint formulas are proposed for standard illuminant D65, one each for the CIE 1931 2° standard observer:

$$T = -1000(x - x_0) + 700(y - y_0),$$

and for the CIE 1964 10° supplementary standard observer:

$$T = -900(x - x_0) + 800(y - y_0).$$

Whiteness formulas of the BGA type with equivalent properties are presented for use with colorimeters.

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